AP Calculus AB — Homework 0224 — Name:

Dr. Paul L. Bailey — Monday, February 24, 2025

Due Tuesday, February 25, 2025.

Write your homework *neatly*, *in pencil*, on blank white $8\frac{1}{2} \times 11$ printer paper. Always *write the problem*, or at least enough of it so that your work is readable. If the problem involves a function, write the function. If the problem involves an equation, write the equation. Use words, and when appropriate, *write in sentences.*

An interval is *maximal* with respect to a condition if it is not a proper subset of another interval which satisfies the condition. That is, if it is not contained in a bigger interval which also satisfies the condition.

An *interior point* of an interval is a point in the interval which is not an endpoint.

The phrase "find and classify the critical points of a function f" means:

- Find all interior points x in the domain of f such that $f'(x) = 0$ or $f'(x)$ does not exist.
- Determine whether each critical point gives a local maximum, a local minimum, or neither.

Problem 1 (1997BC.MC.NC.3)**.** Let

$$f(x) = 3x^5 - 4x^3 - 3x.$$

Find and classify the critical points of f.

Problem 2 (1997AB.MC.NC.5)**.** Let

$$f(x) = 3x^4 - 16x^3 + 24x^2 + 48.$$

(a) Find f''.

(b) Solve $f''(x) = 0$ and create a sign chart for f''.

(c) Identity maximal intervals on which f is concave up or concave down.

Problem 3 (Thomas §4.5 # 4)**.** A rectangle has its base on the x-axis and its upper two vertices on the parabola $y = 12 - x^2$. What is the largest area the rectangle can have, and what are its dimensions?

Problem 4 (Thomas §4.5 # 14)**.** What are the dimensions of the lightest open-top right circular cylindrical can that will hold a volume of 1000cm^3?

Problem 5 (Thomas §4.2 # 5 - 8)**.** Which functions satisfy the Mean Value Theorem on the indicated interval, and which do not? Justify you answer.

(a) $f(x) = x^{2/3}$ on $[-1, 8]$

(b) $f(x) = x^{4/5}$ on $[0, 1]$

(c) $f(x) = \sqrt{x(1-x)}$ on $[0, 1]$

(d) $f(x) = \begin{cases} \dfrac{\sin x}{x} & \text{for } x \in [-\pi, 0) \\ 0 & \text{for } x = 0 \end{cases}$

Problem 6 (Thomas §3.2 # 28)**.** Let $y = \dfrac{(x+1)(x+2)}{(x-1)(x-2)}$. Compute $\dfrac{dy}{dx}$.

Problem 7 (Thomas §5.6 # 21)**.** Compute

$$\int_0^1 \frac{12y^2 - 2y + 4}{\sqrt[3]{(4y - y^2 + 4y^3 + 1)^2}}\, dy.$$

Problem 8. Create an example of a function which is differentiable on $\mathbb{R}$ and whose derivative is not differentiable on $\mathbb{R}$.

Problem 9. Create an example of a function $f : \mathbb{R} \to \mathbb{R}$ which is increasing everywhere yet has infinitely many points of inflection.

Fact 1. Recall the quadratic formula: if $f(x) = ax^2 + bx + c = 0$, then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

The *discriminant* of f is

$$\Delta = b^2 - 4ac.$$

Then

- if $\Delta > 0$, f has exactly two real zeros.
- if $\Delta = 0$, then f has exactly one real zero.
- if $\Delta < 0$, then f has no real zeros.

Use this basic fact to solve the following problem.

Problem 10. Consider the cubic polynomial

$$f(x) = x^3 + ax^2 + bx.$$

Since f is a polynomial of odd degree, f has at least one real zero.

(a) Find the values of a and b for which f has exactly three zeros.

(b) Find the values of a and b for which f has exactly two zeros.

(c) Find the values of a and b for which f has exactly one zero.

(d) Find the values of a and b for which f has exactly two local extrema.

(e) Find the values of a and b for which f has exactly one horizontal tangent.

(f) Find the values of a and b for which f has no horizontal tangents.